NONLINEAR DYNAMICS OF INTELLECTUAL RESOURCES OF SCIENCE AND TECHNOLOGY POTENTIAL AND VACANCIES IN R&D SYSTEM

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One of the main problems of modern Scientology is the study of the state of employment in research and development (R&D) sphere. This problem is topical for all countries but special attention should be paid to this issue in transition economies, especially for Central and Eastern Europe. Sharp diminution of state financing of R&D sphere in this countries caused reduction of the number of research institutions, considerable decay of material and technical basis, reduction of wages, decreasing of demand of scientific activity results, setback in production, lowering of competitiveness of goods on internal and external markets, descent of the prestige of researchers. All these consequences resulted in massive internal and external brain drain of scientists, engineers and technical personnel. The continuing tendency to aging of human scientific potential is another serious problem. All these issues make the problem of employment in R&D sphere of primary importance.

One of the new approaches for study of nonlinear dynamics of economic systems, including intellectual resources of S&T potential and vacancies in R&D sphere, is the synergetics founded basically on nonlinear theory of differential equation, bifurcation theory, theory of dynamic chaos and catastrophe theory [1-9].

The problem of matching of dynamics of vacancies and labour power, assessment of the imbalance of supply and demand of labour was presented in several papers [10-12].

For studying nonlinear dynamics of S&T intellectual potential and vacancies the theory of dynamics of vacancies and labour is used.

If $N(t)$ denotes the number of human that are able to work in R&D

sphere, $L(t)$ - the number of engaged in R&D sphere in the year t, then $n(t) = N(t) - L(t)$ denotes the number of potential workers in R&D area.

Let $w_I(t)$ denotes number of vacancies, in the field of science i, in the year t, $\alpha_i(t)$ - rate of opening new vacancies in R&D sphere, in the filed of science i, in the year t; $\beta_i(t)$ - rate of closing workplaces in R&D area, in the filed of science i, in the year t ; $Z_i(t)$ - number of workers that dropped out of R&D sphere, in the filed of science i, in the year t; $Y_i(t)$- number of new engaged in R&D sphere, in the filed of science i, in the year t. As fields of sciences here we understand academic science, university science, ministerial science, industrial science.

Thus, the dynamics of the number of vacancies will be defined by the following differential equation:

$$\frac{dw_i}{dt} = [\alpha_i(t) - \beta_i(t)]w_i + Z_i(t) - Y_i(t), \; i = 1,2...\overline{n} \qquad (1).$$

The explicit form of the number of newcomers in the field of science i or the function of employment can be different. In particular if we apply the predator-prey model, used in physics, mathematics, chemistry, ecology and economy, supposing that supply of human scientific and technical-engineering potential for vacancies in relevant fields of science are connected by the following equation:

$$Y_i(t) = P_i(t)w_i(t)n(t), \; i = 1,2...\overline{n} \qquad (2)$$

where $P_i(t)$denotes the rate of transaction from potential workers into engaged, and

$$Z_i(t) = q_i(t)Y_i(t), \qquad (3)$$

where $q_i(t)$ is the coefficient denoting the number of employed and discharged in the filed of science i, in the year t .

for $n(t)$ and $w_i(t)$ the following system of nonlinear differential equations is get, which denotes the dynamic evolution of the number of potential employees and vacancies.

$$\frac{dn(t)}{dt} = w(t) - P_0'n(t) + \sum_{i=1}^{\overline{n}} P_i(t)[q_i(t) - 1]w_i(t)n(t)$$
$$\frac{dw_i}{dt} = [\alpha_i(t) - \beta_i(t)]w_i(t) + P_i'(t)[q_i(t) - 1]w_i(t)n(t) \qquad (4)$$

Let suppose that the entrance from outside into the system of employment is determined by relation
$w(t) = P_0''(t)n(t)$ and denoting

$$\varepsilon_0(t) = P_0''(t) - P_0'(t),\ \varepsilon_i(t) = \alpha_i(t) - \beta_i(t),\ \mu_i(t) = P_i'(t)[q_i(t) - 1],$$

we obtain

$$\frac{dn(t)}{dt} = \left[\varepsilon_0(t) + \sum_{i=1}^{\bar{n}} \mu_i(t) w_i(t)\right] n(t),\quad \frac{dw_i}{dt} = [\varepsilon_i(t) + \mu_i(t) n(t)] w_i(t). \quad (5)$$

This system of equations is the main principle for studying nonlinear dynamics of the number of potential employees and vacancies in the R&D area.

It should be mentioned that the dynamics of the number of potential employees and vacancies in R&D sphere determined by the system of nonlinear equations (5) depends considerably on the correlation of values and signs of coefficients $\varepsilon_0(t)$, $\varepsilon_i(t)$, $\mu_i(t)$, which in general case depend on the time. The coefficient of increment of potential employees $\varepsilon_0(t)$ refers to the processes of migration, number of postgraduates, doctorate and other factors. The coefficients $\varepsilon_i(t)$ that denote the increase of vacancies vary because of the opening new vacancies or cutting jobs in R&D sphere. The coefficients $\mu_i(t)$ represent the intensity of the general movement of the labour force and movement of vacancies and vary depending on employment restrictions and movement of labour force.

For investigation of the problem related to prediction of the number of vacancies and potential employees in R&D sphere it is necessary to have the prediction of parameters $\varepsilon_0(t)$,$\varepsilon_i(t)$, $\mu_i(t)$, which is the subject of another investigation.

Stationary solutions and investigation of their stability

As far as variables $n(t)$ and $w_i(t)$ have determined economic sense (number of potential employees and number of vacancies) they can not be negative values. In this case Euclidean plane (*n, w*) represents positive quadrant of the right half-plane $n \geq 0$ and $w \geq 0$.

For simplicity let first investigate one-sector model of the science, supposing that $i=1$. In this case the system of equation (5) will be following:

$$\frac{dn}{dt} = \varepsilon_0(t)n + \mu(t)wn,\ \frac{dw}{dt} = \varepsilon_1(t)w + \mu(t)nw. \quad (6)$$

As it was mentioned coefficients ε_0, ε_i and μ are functions of time. They vary in the period of aggravation of structural disproportions in S&T area. Since the periods of system evolution in general are longer than the periods of functional disproportions generation it can be supposed that these coefficients do not depends on the time during very long periods.

Let introduce the following values:

$$T = |\varepsilon_0|(t),\ \nu = \frac{\varepsilon_0}{|\varepsilon_0|} = \pm 1. \quad (7)$$

In this case the system of equations (6) will be as follows:

$$\frac{dn}{dT} = \nu n + \frac{\mu}{|\varepsilon_0|} w + \frac{\mu}{|\varepsilon_0|} nw,\ \frac{dw}{dT} = \frac{\varepsilon_1}{|\varepsilon_0|} w + \frac{\mu}{|\varepsilon_0|} nw. \quad (8)$$

Determination of accurate analytical solutions of the system of equations (8) is a difficult task, if feasible, because of the absence of solution algorithms of the system of nonlinear differential equations. Though it admit stationary solution when $\frac{dn}{dT} = \frac{dw}{dT} = 0$. One of the stationary solution can be trivial one, when $\bar{n}_{st} = w_{st} = 0$. Another trivial solution is following:

$$n_{st} = -\frac{\varepsilon_1}{\mu},\ w_{st} = -\nu\frac{|\varepsilon_0|}{\mu}. \quad (9)$$

Because the values n_{st} and w_{st} are positive, this fact determines the correlation between coefficients' signs ε_0, ε_1 and μ. If $\varepsilon_0 > 0$ then the meeting of the condition of positivity of stationary states $n_{st} > 0$, $w_{st} > 0$ will result in $\nu = 1$, $\mu < 0$, and $\varepsilon_1 > 0$.

In this case the system of equation (8) will have the following form:

$$\frac{dn}{dT} = n - \bar{\mu} nw \, , \; \frac{dw}{dT} = \bar{\varepsilon}_1 w - \bar{\mu} nw \, , \tag{10}$$

where $\bar{\mu} = \frac{|\mu|}{\varepsilon_0}$, $\bar{\varepsilon}_1 = \frac{\varepsilon_1}{\varepsilon_0}$, $n_{st} = \frac{\varepsilon_1}{|\mu|}$, $w_{st} = \frac{\varepsilon_0}{|\mu|}$.

Thereby the system of equations (10) has two special points that correspond to the stationary state and $(\frac{\varepsilon_1}{|\mu|}, \frac{\varepsilon_0}{|\mu|})$.

If the system of equations (10) is in the state of equilibrium, then the affix is immobile in one of the stationary states on the phase plan. In case the system is took out from state of equilibrium the affix begins to move through phase plan in concordance with system of equation (10).

Of a special interest is the stability of stationary states on the Poincare and Lyapunov, which determines the character of affix's movement in case of deviation from the state of equilibrium. Let ξ and η determine small shift related to the state of equilibrium. Then for stationary point $(0,0)$ and neglecting with non-linear terms as ξ and η from the system of equations (10) we get the following:

$$\frac{d\xi}{dT} = \xi \, , \; \frac{d\eta}{dT} = \varepsilon_1 \eta \, . \tag{11}$$

The solutions of the system of equations (11) have the following form:

$$\xi = Ae^{T} \, , \eta = Be^{\varepsilon_1 T} \, .$$

As far as $\varepsilon_1 > 0$ is real value, the stationary point $(0,0)$ is unstable node.

Let introduce for stationary point $(\frac{\varepsilon_1}{|\mu|}, \frac{\varepsilon_0}{|\mu|})$ new values:

$$n = n_{st} + \xi \, , \; w = w_{st} + \eta \, . \tag{12}$$

After substitution of the formula (12) in the system of equation (10) and linearization of the system of equation neglecting with non-linear terms we get the following formula:

$$\frac{d\xi}{dT} = -\frac{\varepsilon_1}{|\varepsilon_0|} \eta \, , \; \frac{d\eta}{dT} = -\xi \, . \tag{13}$$

The solution of the system of equation (13) is presented as follows:

$$\xi = Ae^{KT} \, , \eta = Be^{KT} \, . \tag{14}$$

When substituting the formulas (14) in (13) we get a system of algebraic equations for determining the constants A and B with the following form:

$$kA + \frac{\varepsilon_1}{|\varepsilon_0|} B = 0 \, , \; A + kB = 0 \, . \tag{15}$$

Non-zero solution for A and B is possible if the determinant composed from coefficients by unknown variables equal zero:

$$\begin{vmatrix} k & \frac{\varepsilon_1}{|\varepsilon_0|} \\ 1 & k \end{vmatrix} = 0 \, . \tag{16}$$

After opening of the determinant (16) we get the following form:

$$k^2 = \frac{\varepsilon_1}{|\varepsilon_0|} \, , \tag{17}$$

from which we obtain two values k_1, k_2 for k and nontrivial solution for the system of equation (15):

$$k_1 = +\sqrt{\frac{\varepsilon_1}{|\varepsilon_0|}} \, , k_2 = -\sqrt{\frac{\varepsilon_1}{|\varepsilon_0|}} \, . \tag{18}$$

General solution for the system of equations (13) is the linear combination of the exponents with values k_1 and k_2:

$$\xi = c_1 e^{KT} + c_2 e^{K_2 T} \, , \eta = c_3 e^{K_1 T} + c_4 e^{K_2 T} \, . \tag{19}$$

As soon as the roots k_1 and k_2 are real and of different signs, the behaviour of the variables form on the phase plan the hyperbolic curves, while the special point is $(\frac{\varepsilon_1}{|\mu|}, \frac{\varepsilon_0}{|\mu|})$ unstable saddle. Thus, both stationary points $(0,0)$ and $(\frac{\varepsilon_1}{|\mu|}, \frac{\varepsilon_0}{|\mu|})$ are unstable. This means that the equations (10) do not admit stationary states and the investigated system evolves in time in a nonlinear form.

It is of interest to find the integral of movement for differential system of equations (10).

We exclude dT from the system of equations (10), and get:

$$d\frac{dn}{dw}=\frac{n-\frac{|\mu|}{\varepsilon_0}nw}{\frac{\varepsilon_1}{\varepsilon_0}w-\frac{|\mu|}{\varepsilon_0}nw}, \qquad (20)$$

Let present the expression (20) in the following form:

$$\frac{\varepsilon_1}{\varepsilon_0}\frac{dn}{n}-\frac{|\mu|}{\varepsilon_0}dn=\frac{dw}{w}-\frac{|\mu|dw}{\varepsilon_0} \qquad (21)$$

After integrating (21) we obtain:

$$\ln\frac{n^{\frac{\varepsilon_1}{\varepsilon_0}}}{w}-\ln c=\frac{|\mu|}{\varepsilon_0}(n-w), \qquad (22)$$

Where c is integrating constant or integral of movement.

If n^0 and w^0 - starting values for variables n_1 and w_1 then from (22) it is easy to get integral of movement:

$$c=\frac{(n_1^0)^{\frac{\varepsilon_1}{\varepsilon_0}}e^{-\frac{|\mu|}{\varepsilon_0}n^0}}{w^0e^{-\frac{|\mu|}{\varepsilon_0}w^0}}, \qquad (23)$$

In dependence of given conditions, integral of movement can take different values c_i. When values c_i are known we can find dependence of n_1 from n_2 from expression:

$$c=\frac{n^{\frac{\varepsilon_1}{\varepsilon_0}}e^{-\frac{|\mu|}{\varepsilon_0}n}}{we^{-\frac{|\mu|}{\varepsilon_0}w}}, \qquad (24)$$

Under diverse values of ε_0, ε_1 and μ.

It should be said, that on study values – phase trajectories are hyperbolical and do not have closed curves. This leads to conclusion that number of potential working places and vacancies do not have periodical solutions.

We have made quantitative experiment due to impossibility of analytical solution for nonlinear differential system.

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