Fuzzy multicriterial optimizations in the transportation problem

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Abstract

In the paper an iterative fuzzy programming approach for solving the multi-objective transportation problem of "bottleneck" type with some imprecise data is developed. Minimizing the worst upper bound to obtain an efficient solution which is close to the best lower bound for each objective function iterative, we find the set of efficient solutions for all time levels.

Keywords: fuzzy programming, fuzzy model, transportation problem, efficient solution.

1 Introduction

It's well known, the increasing of criteria number and imposing of minimal time for realizing the model solution leads only to increasing of solution accuracy for optimal decision making problems. There are many efficient algorithms that solve such models with deterministic data [2]. Since in real life, some parameters are often of fuzzy type, in the proposed work this case is studied.

2 Problem formulation

Because in any optimization model, objective function coefficients have the largest share in the objective function variations, we shall consider these of fuzzy type and develop the next multi- criteria transportation problem of "bottleneck" type with fuzzy costs coefficients:

$$\min Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^{1} x_{ij} \quad \min Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^{2} x_{ij}$$

$$\ldots\ldots\ldots$$

$$\min Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^{r} x_{ij} \quad \min Z_{r+1} = \max_{i,j} \{ t_{i,j} | \, x_{i,j} > 0\} \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \forall i = \overline{1,m}, \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad \forall j = \overline{1,n},$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \quad x_{ij} \geq 0 \; for \; all \;\; i \; and \; j,$$

where : $\tilde{c}_{ij}^{k}$, $k = 1, 2 \ldots r, i = 1, 2, \ldots m, j = 1, 2, \ldots n$ are costs or other amounts of fuzzy type, t_{ij} – necessary unit transportation time from source i to destination j, a_i – disposal at source i, b_j – requirement of destination j, x_{ij} – amount transported from source i to destination j.

In the model there may exist the criteria of maximum too, which however does not complicate it.

3 Theoretical analysis of fuzzy cost multi-criteria transportation model

Since the parameters and coefficients of transportation multi-criteria models have real practical significances such as unit prices, unit costs and many other, all of them are interconnected with the same parameter of variation, which can be calculated by applying various statistical methods. We propose to calculate it using the following formula:

$$p_{ij}^{k} = \frac{c_{ij}^{k} - \underline{c}_{ij}^{k}}{\bar{c}_{ij}^{k} - \underline{c}_{ij}^{k}}, \tag{2}$$

where: $\underline{c}_{ij}^{k}$, $\bar{c}_{ij}^{k}$ – are the limit values of variation interval for each cost coefficient c_{ij}^{k}, where: $i = \overline{1,m},\ j = \overline{1,n},\ k = \overline{1,r}$.

Agreeing to the formula (2), the parameters $\{p_{ij}^{k}\}$ can be considered as the probabilistic parameters of belonging for every value of coefficients $\{c_{ij}^{k}\}$ from their corresponding variation intervals.

The main idea of the method that follows, is the simultaneous and interconnected variation of objective functions coefficients. This makes it possible to reduce the model (1) to a set of deterministic models that can be solved by applying the fuzzy techniques [1].

4 Some reasoning and algorithms

Seeing that the model (1) is of multi-criteria type, for its solving usually it builds a set of efficient solutions, known also as Pareto-optimal solutions. Since solving model (1) involves its iterative reducing to some deterministic we should propose firstly the following definitions.

Let us suppose that: $(\bar{\mathrm{X}}, \bar{\mathrm{T}})$ is one basic solution for the model (1), where: $\bar{\mathrm{T}} = \max\limits_{i,j}\{\bar{t}_{ij}/\bar{x}_{ij} > 0\}$ *and* $\overline{X} = \{\overline{x_{ij}}\}$, $i = \overline{1,m}$, $j = \overline{1,n}$ *is one basic solutions for the first* r *– criteria model* (1).

Definition 1. *The basic solution* $(\bar{\mathrm{X}}, \bar{\mathrm{T}})$ *of the model (1) is a basic efficient one if and only if for any other basic solution* $(X, T) \neq (\bar{X}, \bar{T})$ *for which exists at least one index* $j_1 \in (1, ...r)$ *for which the relation* $Z_{j_1}(X) \leq Z_{j_1}(\bar{X})$ *is true, there immediately exists another, at least, one index* $\exists j_2 \in (1, ...r)$, *where* $j_2 \neq j_1$, *for which at least, one of the both relations* $Z_{j_2}(\bar{X}) < Z_{j_2}(X)$ *or* $\bar{T} < T$ *is true. If all of these three inequalities are verified simultaneously with the equal sign, it means that the solution is not unique.*

Definition 2. *The basic solution* $(\bar{\mathrm{X}}, \bar{\mathrm{T}})$ *of the model (1) is one of the optimal (best) compromise solution for a certain time* $\bar{T}$, *if the solution* $\bar{\mathrm{X}}$ *is located most closely to the optimal solutions of each criterion.*

In order to solve deterministic model (1) we can use the ***fuzzy technique*** [1] and iteratively solve the deterministic model (3) for the best - L_k and the worst $U_k$values of k-criterion.

Max λ *in the same availability conditions as in* (1) *and:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} + \lambda \cdot (U_k - L_k) \leq U_k, \quad k = \overline{1, r}, \tag{3}$$

By iterative applying the fuzzy technique for each increasing time level, we could get the set of all its optimal compromise solutions.

5 Conclusion

By applying the hypothesis about the interconnection and similarly variation of the model's objective functions coefficients, we reduce the model (1) to several models of deterministic type, each of which may be solved using fuzzy technique.

References

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