Parallel algorithm to find the Bayes-Nash solution in the informational extended game

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Abstract

The Bayes-Nash solutions for informational extended games are discussed. Also the parallel algorithm for mixed system with shared and distributed memory to determine the Bayes-Nash solutions in the bimatrix informational extended games are presented.

Keywords: games, strategies, Bayes-Nash solution, parallel algorithm.

1 Informational extended game

We consider the perfect and complete bimatrix game in strategic form $\Gamma = \langle I, J, A, B\rangle$. According to [1] we can describe the informational extended strategies in bimatrix game as follows. For all fixed $\alpha = 1, ..., n^m$ and $\beta = 1, ..., m^n$ we construct the vectors $\mathbf{i}^\alpha = \left(i_1^\alpha, i_2^\alpha, ..., i_j^\alpha, ..., i_m^\alpha\right)$ and $\mathbf{j}^\beta = \left(j_1^\beta, j_2^\beta, ..., j_i^\beta, ..., j_n^\beta\right)$. The $\mathbf{i}^\alpha$ vector's elements mean the following: if the player 2 will choose the column $j \in J$, then the player 1 will choose the line $i_j^\alpha \in I$. Respectively, the $\mathbf{j}^\beta$ vector's elements mean the following: if the player 1 will choose the line $i \in I$, then the player 2 will choose the column $j_i^\beta \in J$. So we can introduce the following definition. Denote by $I^\alpha = \left\{i_j^\alpha \in I : i_j^\alpha \neq i_k^\alpha, \forall j, k \in J, j \neq k\right\}$ and $J^\beta = \left\{j_i^\beta \in J^\beta : j_i^\beta \neq j_r^\beta \forall i, r \in I, i \neq r\right\}$. Then the set $I^\alpha \subseteq I$, respectively $J^{\beta} \subseteq J$, is the set of informational non extended strategies of the player 1, respectively 2, generated by the informational extended strategies $\mathbf{i}^{\alpha}$, respectively $\mathbf{j}^{\beta}$. Denote by $G\left(1 \overset{\inf}{\leftrightarrows} 2\right)$ the bimatrix game in the informational extended strategies, described above. This game is in the imperfect information on the set of informational extended strategies. According to [2], for bimatrix game $G\left(1 \overset{\inf}{\leftrightarrows} 2\right)$ we construct the Selten-Harsanyi [3] type normal form game $\Gamma^{*}_{Bayes} = \left\langle K, \{R_k\}_{k\in K}, \{U_k\}_{k\in K}\right\rangle$. Here the set of type-players is $K = K_1 \cup K_2$, where $K_1 = \{\alpha = 1, ..., n^m\}$ and $K_2 = \{\beta = 1, ..., m^n\}$; the sets of pure strategies of the type-players are $R_k = \begin{cases} I^{\alpha} & k \in K_1, \\ J^{\beta} & k \in K_2. \end{cases}$; the payoff functions of the type-player k are $U_k = \begin{cases} \mathcal{A}_k\left(\{p(\beta/\alpha)\}_{\beta\in K_2}\right) & k \in K_1, \\ \mathcal{B}_k\left(\{q(\alpha/\beta)\}_{\alpha\in K_1}\right) & k \in K_2, \end{cases}$ where $\mathcal{A}_k\left(\{p(\beta/\alpha)\}_{\beta\in K_2}\right)$ and $\mathcal{B}_k\left(\{q(\alpha/\beta)\}_{\alpha\in K_1}\right)$ are the payoff matrixes of the type-players. In the other words for all fixed "believer probabilities" $p(\beta/\alpha)$ and $q(\alpha/\beta)$, the payoff matrix for the type-players $k \in K_1$ is $\mathcal{A}_k\left(\{p(\beta/\alpha)\}_{\beta\in K_2}\right) = \|\widetilde{a}_{ij}\|_{i\in I}^{j\in J}$, where $\widetilde{a}_{ij} = \sum\limits_{\beta\in K_2} p(\beta/\alpha) a_{i_j^{\alpha} j_i^{\beta}}$, and for the type-players $k \in K_2$ is $\mathcal{B}_k\left(\{q(\alpha/\beta)\}_{\alpha\in K_1}\right) = \left\|\widetilde{b}_{ij}\right\|_{i\in I}^{j\in J}$, where $\widetilde{b}_{ij} = \sum\limits_{\alpha\in K_1} q(\alpha/\beta) b_{i_j^{\alpha} j_i^{\beta}}$. Here $i_j^{\alpha} \in I^{\alpha}$ and $j_i^{\beta} \in J^{\beta}$. The Selten-Harsanyi game Γ^{*}_{Bayes} means the following: for all fixed type-players k_1 and k_2 (i.e. the player 1 chooses the informational extended strategy α and the player 2 chooses the informational extended strategy β) and "believer probabilities" $\{p(\beta/\alpha)\}_{\beta\in K_2}$ of the player 1 (respectively $\{q(\alpha/\beta)\}_{\alpha\in K_1}$ of the player 2) we obtain the bimatrix game $\Gamma^{*} = \left\langle I, J, \mathcal{A}_{k_1}\left(\{q(\alpha/\beta)\}_{\beta\in K_2}\right), \mathcal{B}_{k_2}\left(\{q(\alpha/\beta)\}_{\alpha\in K_1}\right)\right\rangle$ in the non informational extended strategies. So Selten-Harsanyi game Γ^{*}_{Bayes} "generates" the big number of the bimatrix games of type Γ^{*}.

2 Parallel algorithm

According to [2] determining all Bayes-Nash equilibrium profiles, we can determine the all Nash equilibrium profiles for all bimatrix games of type Γ^* in the non extended strategies. So the parallel algorithm to find the all equilibrium profiles consists of the following main steps.

1. Using the MPI programming model, generate the virtual medium of MPI-process communication (MPI Communicator) with linear topology. Root process broadcasts the initial matrices $A = ||a_{ij}||_{i\in I}^{j\in J}$, and $B = ||b_{ij}||_{i\in I}^{j\in J}$ of the bimatrix game $\Gamma = \langle I, J, A, B\rangle$.
2. MPI process with rank k_1 generates the "beliver-probabilities" $p(\beta/\alpha)$ for all β, and MPI process with rank k_2 generates the "beliver-probabilities" $q(\alpha/\beta)$ for all α.
3. Using the MPI programming model and open source library ScaLAPACK-BLACS [4], initialize the processes grid. All fixed MPI processes (α, β) using the OpenMP directives and combinatorial algorithm construct the sets I^α, J^β.
4. MPI process with rank k constructs payoff matrix U_k.
5. Using open source library ScaLAPACK-BLACS, MPI process broadcasts the matrix U_k.
6. All fixed MPI processes, using the OpenMP functions, eliminate from matrix $\mathcal{A}_{k_1}(\cdot)$ and from matrix $\mathcal{B}_{k_2}(\cdot)$ the lines that are strictly dominated in matrix $\mathcal{A}_{k_1}(\cdot)$ and columns that is strictly dominated in matrix $\mathcal{B}_{k_2}(\cdot)$. Finally we obtain the matrices $\widehat{\mathcal{A}}_{k_1}\left(\{p(\beta/\alpha)\}_{\beta\in K_2}\right)$ and $\widehat{\mathcal{B}}_{k_2}\left(\{q(\alpha/\beta)\}_{\alpha\in K_1}\right)$.
7. All fixed MPI processes using the MPI, OpenMP functions, ScaLAPACK routines and existing algorithm, determine all Nash equilibrium profiles in the bimatrix game with matrices $\widehat{\mathcal{A}}_{k_1}(\cdot)$, $\widehat{\mathcal{B}}_{k_2}(\cdot)$ and construct the set of Nash equilibrium profiles in the bimatrix game with matrices $\mathcal{A}_{k_1}(\cdot)$, $\mathcal{B}_{k_2}(\cdot)$.

8. Using ScaLAPACK-BLACS routines, the root MPI process gather the set of Nash equilibrium profiles in the bimatrix game with matrices U_k.

3 Conclusion

For this algorithm a C++ program has been developed using MPI functions, OpenMP directives and ScaLAPACK routines. Program has been tested on the control examples on the Moldova State University HPC cluster. The test results were consistent with theoretical results. In order to determine all sets of Nash equilibrium profiles in bimatrix games generated by information strategies, it is recommended to use exascale HPC systems.

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